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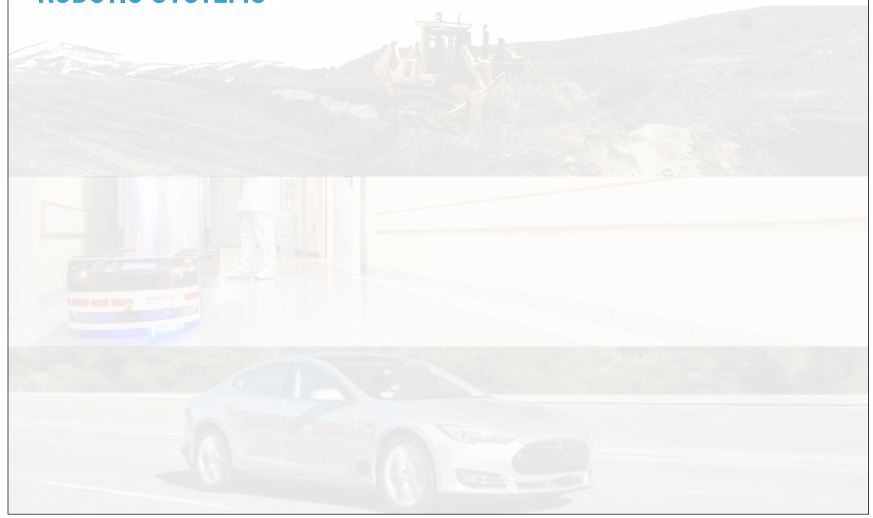
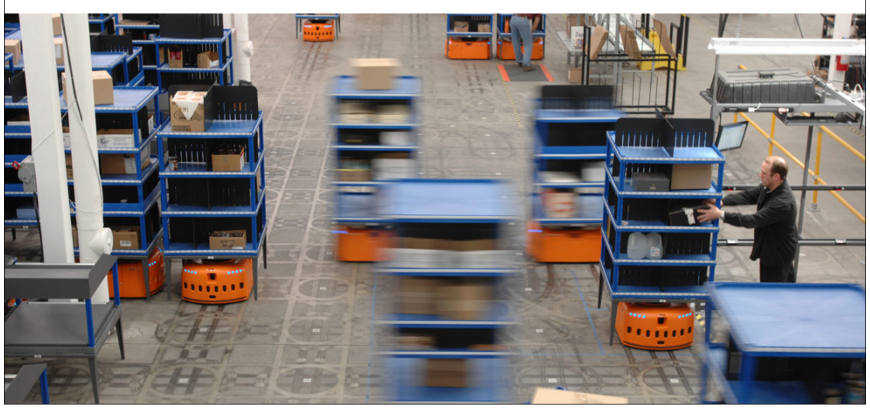
SELECTIVE DETERMINISM FOR AUTONOMOUS NAVIGATION IN MULTI-AGENT SYSTEMS

ROBOTIC SYSTEMS



FUNDAMENTAL CHALLENGES



COMPLEXITY, AI, & PLANNING

- I: Finite set of agents
- S: Finite set of states
- A: Finite set of actions
- T: Transition probability functions
- O: Observation function
- R: Reward function

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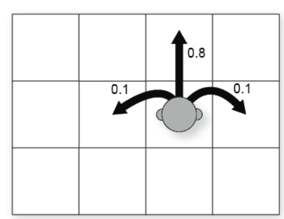
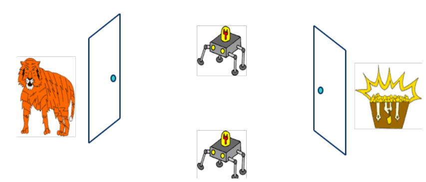




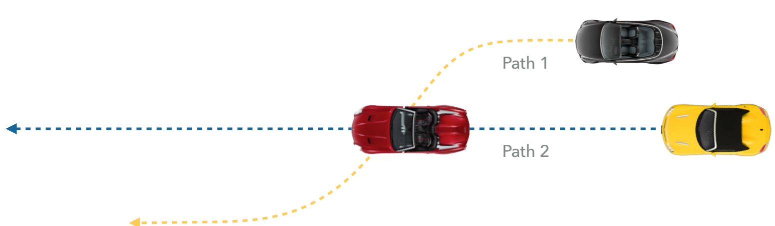
PRIMARY RESEARCH QUESTIONS

1. In a stochastic multi-agent system, precisely how can a multi-agent navigation problem be decomposed into independent sub-problems?
2. Assuming such a decomposition, how can the solutions of those sub-problems be re-combined to provide a solution to the original problem?

OUTLINE OF CONTRIBUTIONS

- Path-time planning
- The constrained interference minimization principle
- Factoring interaction effects in collision avoidance
- Selective Determinism for multi-agent navigation

PATH-TIME PLANNING



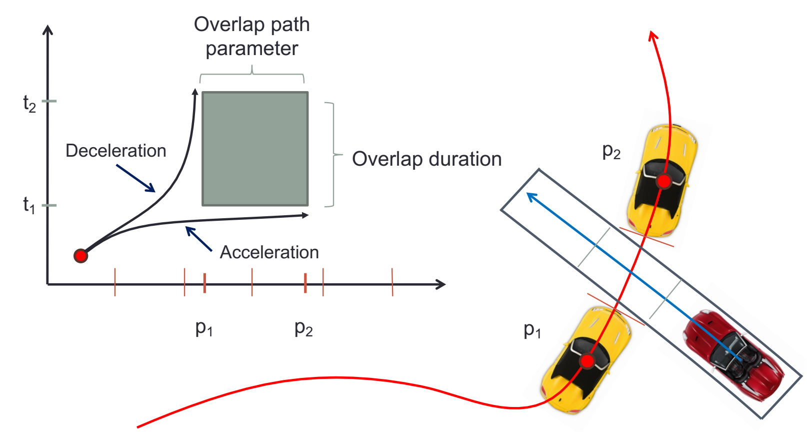
Constraints:

$$\dot{s} \in [\dot{s}_{\min}, \dot{s}_{\max}], \quad \dot{s}_{\min} \geq 0$$
$$\ddot{s} \in [\ddot{s}_{\min}, \ddot{s}_{\max}], \quad \ddot{s}_{\min} < 0 < \ddot{s}_{\max}$$

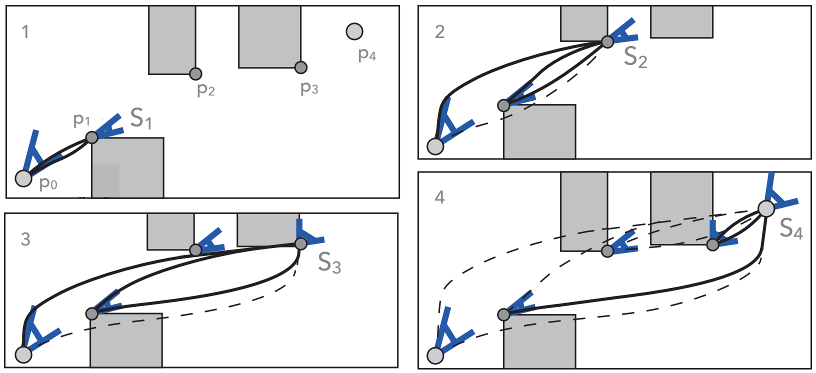
Assumptions:

- Deterministic agent policies
- Path-independent constraints

PATH-TIME PLANNING: PATH-TIME SPACE

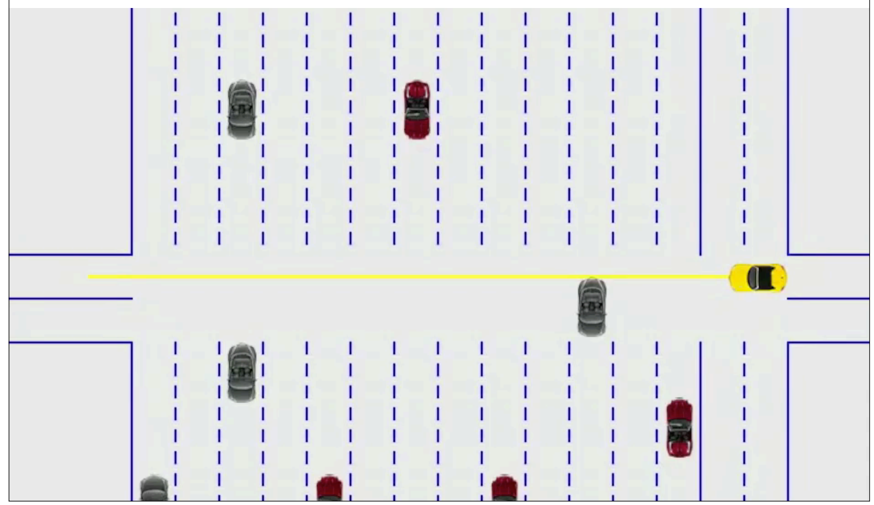


PATH-TIME PLANNING: VISIBILITY GRAPH CONSTRUCTION



Intuition: Tangencies bound reachable regions
Approach: Iteratively compute reachability at tangencies

PATH-TIME PLANNING: DEMONSTRATION



CONSTRAINED INTERFERENCE MINIMIZATION

- For an input control, compute the nearest output control that maintains a desired property with a given confidence

$$\mathbf{u}_t^\star = \arg\min_{\mathbf{u}} \mu(\mathbf{u}, \mathbf{u}_t^d)$$
$$\text{s.t.} \quad P(\text{good} \mid \mathbf{u}_t = \mathbf{u}) \geq \alpha$$

CONSTRAINED INTERFERENCE MINIMIZATION: COMPUTATION

- Unfortunately, many practical systems are difficult to solve (e.g. do no exhibit certainty equivalence)
- What about approximation techniques?
- The rollout method:

CONSTRAINED INTERFERENCE MINIMIZATION

- Back to the original problem:

$$\mathbf{u}_t^\star = \arg\min_{\mathbf{u}} \mu(\mathbf{u}, \mathbf{u}_t^d)$$
$$\text{s.t.} \quad P(\text{good} \mid \mathbf{u}_t = \mathbf{u}) \geq \alpha$$

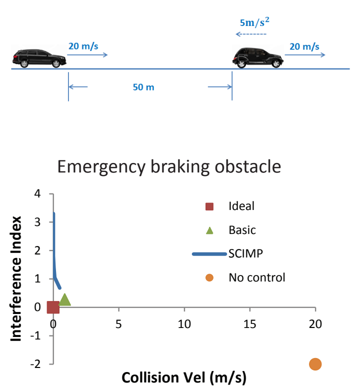
CONSTRAINED INTERFERENCE MINIMIZATION

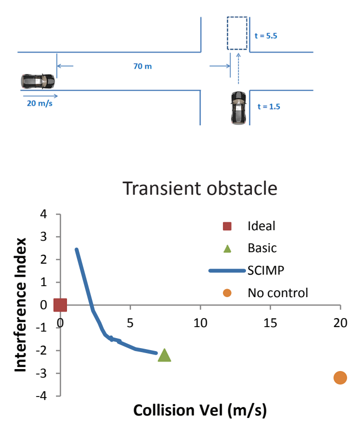
$$P(\text{good} \mid \mathbf{u}_t = \mathbf{u}) \approx \int_{\mathbf{z}} S(\mathbf{x}, \mathbf{u}) p(\mathbf{x})$$

Indicator/
deterministic control problem

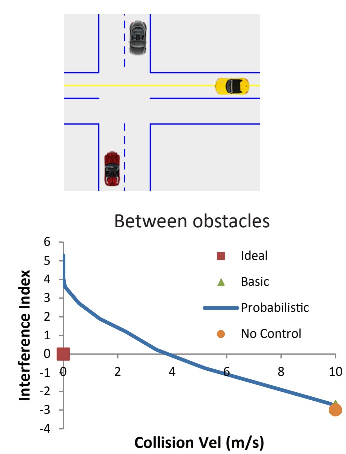
- Under Bayesian interpretation, Monte Carlo integration provides rigorous confidence bounds

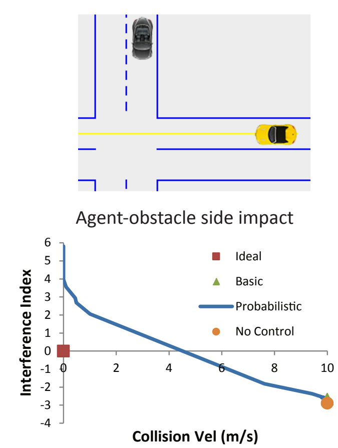
SAFETY-CONSTRAINED INTERFERENCE MINIMIZATION



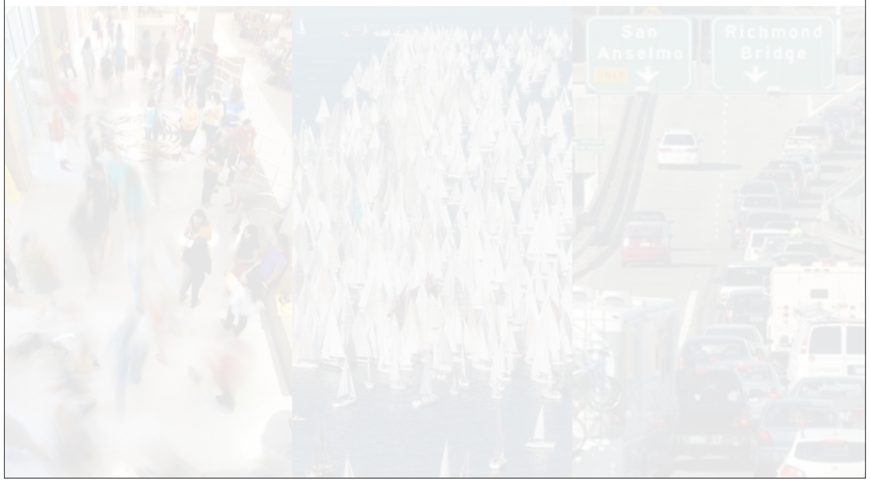


SAFETY-CONSTRAINED INTERFERENCE MINIMIZATION





DYNAMICS AND COMPLEXITY: MOTIVATION

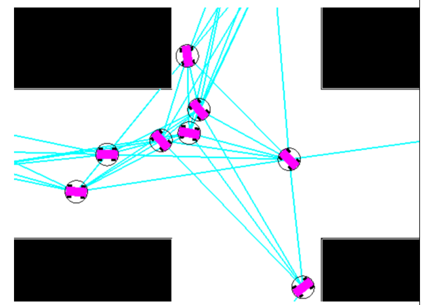


DYNAMICS AND COMPLEXITY: THE COORDINATION PROBLEM

Un-coordinated planning:
Reciprocal *n*-body collision avoidance

Jur van den Berg, et al.

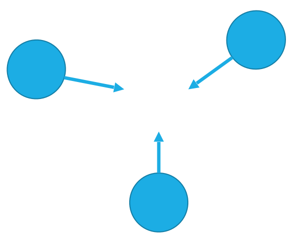
Coordinated planning:
Safe distributed motion coordination



Kostas E. Bekris, et al.

DYNAMICS AND COMPLEXITY: PROBLEM DESCRIPTION

- > 2 Agents
- Observable dynamic state
- Partially observable intent
- Strictly pairwise communication
- Positive, non-zero communication cost

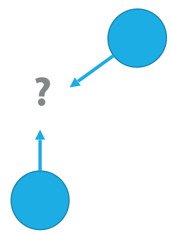


DYNAMICS AND COMPLEXITY: PREMISES

1. **Optimality is not necessary**
   - These problems have no tractable optimal solution
2. **Agents are self-preserving**
   - Practical systems tend not to be demolition derbies
   - Self-preservation generally overwhelms other goals

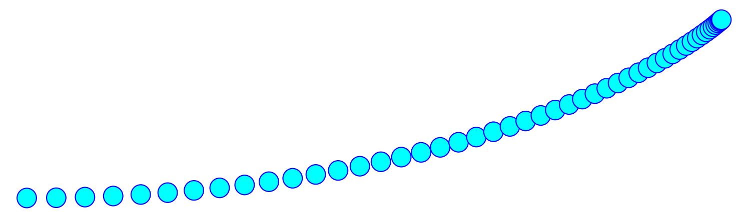
DYNAMICS AND COMPLEXITY: DEFINITIONS

- **Coordination:** The property that the feasibility of two actions cannot be verified independently of each other



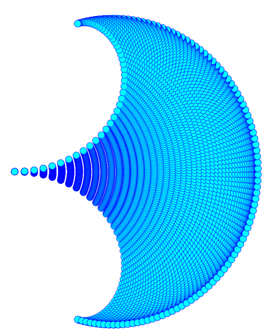
DYNAMICS AND COMPLEXITY: DEFINITIONS

- **Stopping Path (SP):** The minimal set of states an agent must occupy while coming to zero velocity along the path



DYNAMICS AND COMPLEXITY: DEFINITIONS

- **Stopping Region (SR):** The union of all stopping paths over the set of feasible paths



DYNAMICS AND COMPLEXITY: MAIN RESULT

- *A multi-agent system is guaranteed to be able to remain collision free without coordination iff all agents have a SP that is disjoint from all others' SRs.*
- SPs & SRs are an important representation because:
  - They are computable independent of agent intent
  - They can be manipulated by each agent
  - Thus, a system can self-organize away from a coordination requirement

THE SELECTIVE DETERMINISM FRAMEWORK

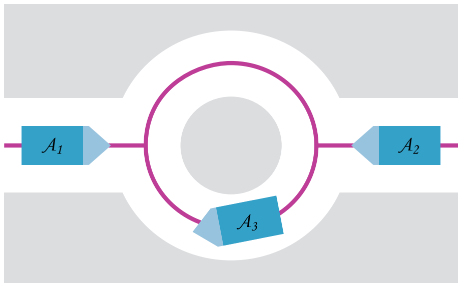
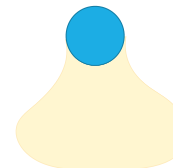


Illustration by Valerie Aquila

- Two Parts: 1) Determine factorization 2) Generate motion control given factorization

DETERMINING A FACTORIZATION

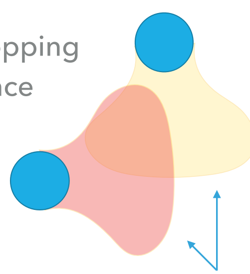
- What is a "factorization"?
  - Set of independence relations
  - Indicated by SP disjointness



Trivially disjoint stopping paths

DETERMINING A FACTORIZATION

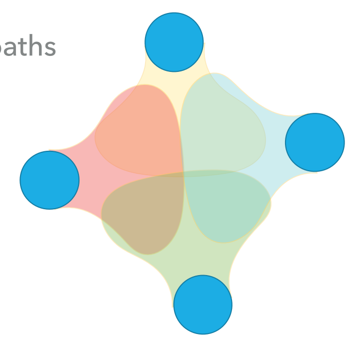
- What is a "factorization"?
  - Set of independence relations
  - The *existence* of disjoint stopping paths indicates independence



Disjoint stopping paths

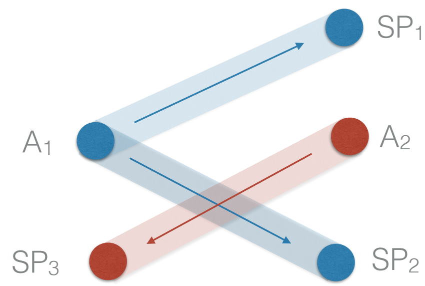
DETERMINING A FACTORIZATION

- What is a "factorization"?
  - Set of independence relations
  - In the absence of disjoint paths coordination is required



DETERMINING A FACTORIZATION

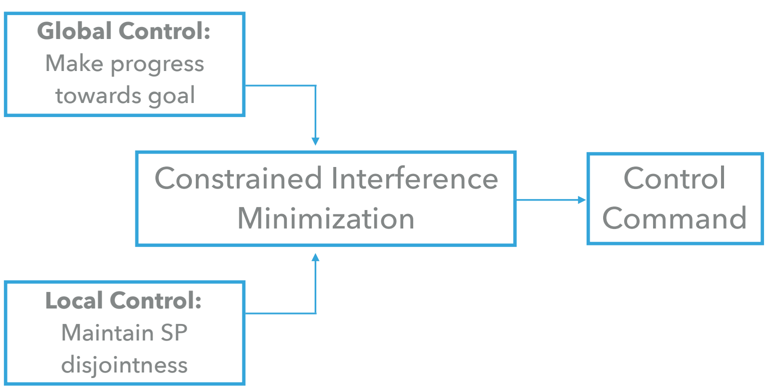
- What is a "factorization"?
  - *Conjecture:* Unique factorizations can also exist without mutually disjoint stopping paths by solving Unique-SAT



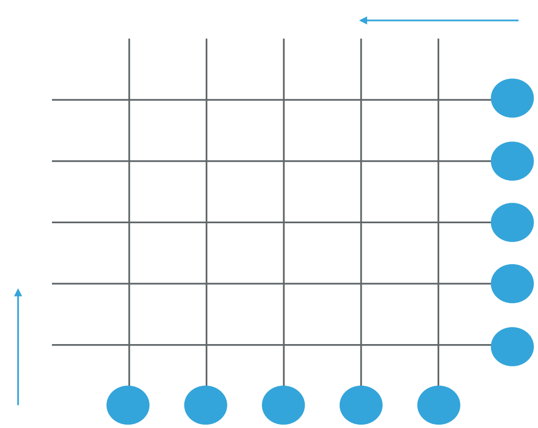
PLANNING UNDER A FACTORIZATION

- Blending of two priorities:
  1. Preserve or seek to re-establish independence
  2. Make progress towards goal
- This is exactly a constrained interference minimization problem

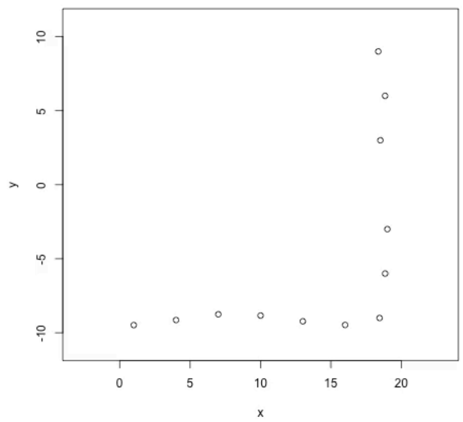
THE SELECTIVE DETERMINISM FRAMEWORK



THE SELECTIVE DETERMINISM FRAMEWORK: EXAMPLE



THE SELECTIVE DETERMINISM FRAMEWORK: EXAMPLE



CONCLUSIONS

- Selective Determinism framework is a principled theoretical framework that allows complex multi-agent navigation problems to be decomposed into separate collision avoidance and goal direction problems
- The framework builds on novel results in control and collision avoidance theory
- The framework is demonstrated using a fully deterministic planner in a partially observable domain

FUTURE WORK

- Different interpretations of SD components
- SP disjointness, particularly, may lend itself to alternative approaches
  - Time-to-contact may provide enough information
  - Would enable non-geometric planning/control spaces
- Extended SP definition (holding patterns, etc.)
- Investigation into Unique-SAT approaches

QUESTIONS?

"How can one have faith in a model predicting that a group of agents will solve an intractable problem?"

Konstantinos Daskalakis, Christos H. Papadimitriou,
The Complexity of Games on Highly Regular Graphs

"…we conclude that control theory has in principle nothing to say about how to explore. It can only compute the optimal controls for future rewards once the environment is known. The issue of optimal exploration is not addressable within the context of optimal control theory. This statement holds for any type of control theory…"

Hilbert J. Kappen,
Optimal Control Theory and the Linear Bellman Equation